

DESY 04-238  
hep-ph/0412162

# Quarks vs. gluons in exclusive $\rho$ electroproduction

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## Abstract

We compare the contributions from quark and from gluon exchange to the exclusive process  $\gamma^* p \rightarrow \rho^0 p$ . We present evidence that the gluon contribution is substantial for values of the Bjorken variable  $x_B$  around 0.1.

PACS numbers: 12.38.Bx, 13.60Le

1. There is an ongoing experimental and theoretical effort to determine generalized parton distributions [1, 2] from hard exclusive processes like deeply virtual Compton scattering and electroproduction of mesons. These distributions encode fundamental information about nucleon structure, in particular about the angular momentum carried by partons [2] and about their spatial distribution [3, 4]. An important process is the production of  $\rho^0$  mesons, well suited for experimental study because of its relatively high cross section and its clean final state signature from the decay  $\rho^0 \rightarrow \pi^+ \pi^-$ . As pointed out in [5], the transverse target polarization asymmetry of this channel is sensitive to the nucleon spin-flip distribution  $E$  appearing in the angular momentum sum rule [2].

Quark and gluon distributions contribute to  $\rho$  production at the same order in  $\alpha_s$ , as seen in Fig. 1. For the separation of quark and gluon degrees of freedom this channel is thus a valuable complement to deeply virtual Compton scattering, which offers the cleanest and most detailed access to generalized parton distributions [6, 7], but is sensitive to gluons only at the level of  $\alpha_s$ .

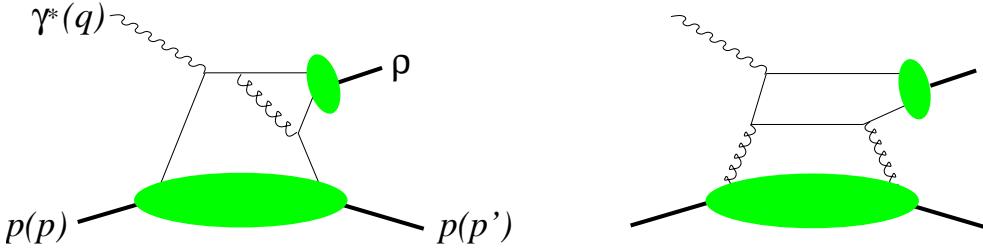


Figure 1: Example graphs for  $\gamma^* p \rightarrow \rho p$  with generalized quark and gluon distributions. Four-momenta are given in parentheses.

corrections. From the behavior of the usual quark and gluon densities one expects  $\rho$  production to be dominated by gluons at very small  $x_B$  and by quarks at very large  $x_B$ , and it is natural to ask where the transition between these two regimes takes place. In this letter we present evidence that quarks and gluons contribute to the  $\rho$  cross section with comparable strength in the  $x_B$  region around 0.1, relevant for measurements at HERMES [8]. Key ingredient in our argument is the measured cross section for  $\phi$  electroproduction, where the gluon distribution should dominate.

2. We consider the exclusive processes  $\gamma^* p \rightarrow \rho p$  and  $\gamma^* p \rightarrow \phi p$  and use the standard kinematic variables  $Q^2 = -q^2$ ,  $W^2 = (p+q)^2$ ,  $x_B = Q^2/(2p \cdot q)$  and  $t = (p-p')^2$ . In the limit of large  $Q^2$  at fixed  $x_B$  and  $t$  the scattering amplitude factorizes into a hard-scattering kernel, generalized quark or gluon distributions, and the light-cone distribution amplitude of the produced meson [9]. We make the approximation that the normalization of the  $\rho$  and  $\phi$  distribution amplitudes is related by  $\langle \rho | \bar{u} \gamma^\mu u - \bar{d} \gamma^\mu d | 0 \rangle = \sqrt{2} \langle \phi | \bar{s} \gamma^\mu s | 0 \rangle$ . This relation leads to a value of 9 : 2 for the ratio  $(M_\rho \Gamma_{\rho \rightarrow e^+ e^-}) : (M_\phi \Gamma_{\phi \rightarrow e^+ e^-})$  of meson mass times partial leptonic width, which compares well with the value 9 : 2.1 from experiment [10]. We further assume that the  $\rho$  and  $\phi$  distribution amplitudes have the same dependence on the quark momentum fraction. The ratio of production amplitudes for the two channels is then<sup>1</sup>

$$\mathcal{A}_\rho : \mathcal{A}_\phi = -\frac{1}{\sqrt{2}} \left( \frac{2}{3} \mathcal{F}^u + \frac{1}{3} \mathcal{F}^d + \frac{3}{4} \mathcal{F}^g \right) : \left( \frac{1}{3} \mathcal{F}^s + \frac{1}{4} \mathcal{F}^g \right) \quad (1)$$

to leading accuracy in  $1/Q$  and in  $\alpha_S$ . Here

$$\begin{aligned}\mathcal{F}^q &= \int_0^1 dx \left[ \frac{1}{\xi - x - i\varepsilon} - \frac{1}{\xi + x - i\varepsilon} \right] \left[ F^q(x, \xi, t) - F^q(-x, \xi, t) \right] \quad (q = u, d, s), \\ \mathcal{F}^g &= \int_0^1 dx \left[ \frac{1}{\xi - x - i\varepsilon} - \frac{1}{\xi + x - i\varepsilon} \right] \frac{F^g(x, \xi, t)}{x} \end{aligned} \quad (2)$$

with  $\xi = x_B/(2 - x_B)$  are the relevant integrals over quark and gluon matrix elements, parameterized by generalized parton distributions as

$$F^q(x, \xi, t) = \frac{1}{(p + p')^+} \left[ H^q(x, \xi, t) \bar{u}(p') \gamma^+ u(p) + E^q(x, \xi, t) \bar{u}(p') \frac{i\sigma^{+\mu}(p' - p)_\mu}{2m} u(p) \right] \quad (3)$$

<sup>1</sup>We remark that there is a mistake in eq. (284) of [7]: in all three terms with  $F^g$  the 8 in the prefactor should be replaced by 4.

for quarks and in analogy for gluons. The distributions are normalized such that in the forward limit and for  $x > 0$  one has  $H^q(x, 0, 0) = q(x)$ ,  $H^q(-x, 0, 0) = -\bar{q}(x)$  and  $H^g(x, 0, 0) = xg(x)$ , for explicit definitions see e.g. [7]. It is understood that the distributions are to be taken at a factorization scale of order  $Q^2$ . We restrict our study to the Born level formulae (1) and (2) and note that at next-to-leading order in  $\alpha_S$  the amplitudes depend in addition on the quark flavor singlet distribution  $\sum_q [F^q(x, \xi, t) - F^q(-x, \xi, t)]$ , which mixes with  $F^g(x, \xi, t)$  under evolution [11].

3. The  $\gamma^* p$  cross section on an unpolarized target involves the combination

$$\frac{1}{2} \sum_{s's} |\mathcal{F}_{s's}|^2 = (1 - \xi^2) |\mathcal{H}|^2 - \left( \xi^2 + \frac{t}{4m^2} \right) |\mathcal{E}|^2 - 2\xi^2 \text{Re}(\mathcal{E}^* \mathcal{H}), \quad (4)$$

where  $s$  and  $s'$  respectively denote the polarization of the initial and final state proton, and where  $\mathcal{F}$ ,  $\mathcal{H}$  and  $\mathcal{E}$  are the relevant linear combinations of integrals over quark and gluon distributions given in (1). In the following we will be interested in kinematics where  $\xi$  is below 0.1 and where the dominant values of  $-t$  are a few times  $0.1 \text{ GeV}^2$ . Both  $\xi^2$  and  $t/(4m^2)$  are then small, so that the term with  $|\mathcal{H}|^2$  will dominate the unpolarized cross section unless  $\mathcal{E}$  is significantly larger than  $\mathcal{H}$ .

There are however no indications from theory or phenomenology that  $|E^q| \gg |H^q|$  or  $|E^g| \gg |H^g|$ . For their lowest  $x$  moments, one has for instance

$$\int_{-1}^1 dx H^u = 2, \quad \int_{-1}^1 dx H^d = 1, \quad \int_{-1}^1 dx E^u \approx 1.67, \quad \int_{-1}^1 dx E^d \approx -2.03, \quad (5)$$

at  $t = 0$ , where the integrals over  $E^q$  are obtained from the anomalous magnetic moments of proton and neutron. A similar situation is seen when comparing the moments  $\int_{-1}^1 dx xH^q$  and  $\int_{-1}^1 dx xE^q$  for  $u$  and  $d$  quarks obtained in lattice QCD [12, 13]. For the gluon distributions, one may even expect that  $E^g$  is relatively small. For the following argument we set  $\xi = 0$  and  $t = 0$ . We then have a sum rule

$$\int_0^1 dx E^g + \sum_q \int_{-1}^1 dx xE^q = 0 \quad (6)$$

from the conservation of momentum and angular momentum (see e.g. [7]). The lattice calculations just cited find that the contributions from  $u$  and  $d$  quarks tend to cancel each other, in line with general considerations from the large  $N_c$  limit of QCD [5]. Depending on how strong this cancellation is, and barring an unexpectedly large contribution from  $s$  quarks to the sum rule (6), the integral  $\int_0^1 dx E^g$  can thus be relatively small compared with the individual quark moments  $\int_{-1}^1 dx xE^u$  and  $\int_{-1}^1 dx xE^d$ . This is in stark contrast to the momentum sum  $\int_0^1 dx H^g = \int_0^1 dx xg(x)$ , which is similar in size to its quark counterparts.

In the following we will thus neglect  $\mathcal{E}$  in the unpolarized cross section (4) because of its kinematic prefactors. We note in passing that the difference of  $\gamma^* p$  cross sections for transverse target polarization above and below the scattering plane is proportional to  $\text{Im}(\mathcal{E}^* \mathcal{H})$ , so that in this observable the contribution of  $\mathcal{E}$  is essential. We observe that with the pattern of relative signs and sizes just discussed, the contributions from  $E^u$  and  $E^d$  in  $\rho$  production will partially cancel according to (1).

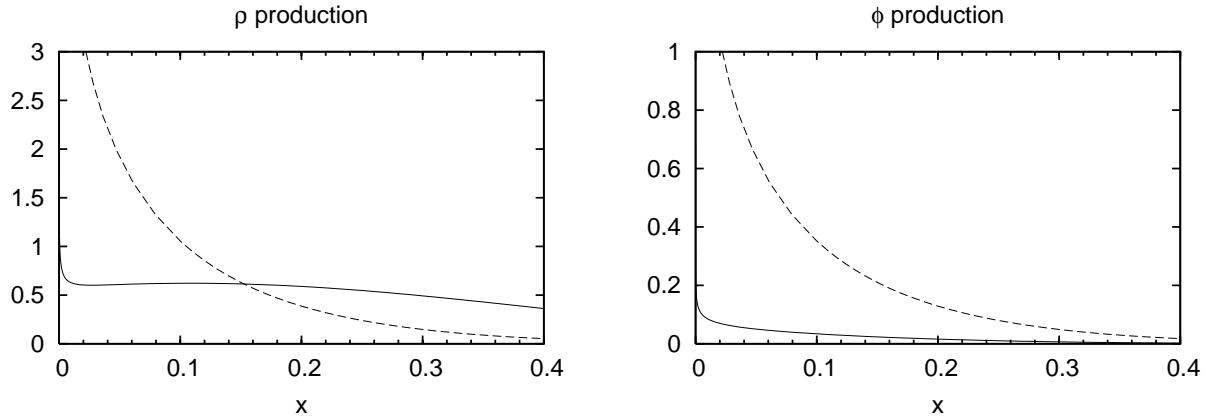


Figure 2: Left: The combinations  $\frac{2}{3}(u + \bar{u}) + \frac{1}{3}(d + \bar{d})$  and  $\frac{3}{4}g$  of parton distributions relevant for  $\rho$  production according to (1) and (2). Right: The combinations  $\frac{1}{3}(s + \bar{s})$  and  $\frac{1}{4}g$  relevant for  $\phi$  production. Full lines correspond to quarks and dashed lines to gluons.

4. For a very rough estimate of the relative importance of the different terms in (1) let us neglect that the dependence of generalized parton distributions on  $\xi$  and on  $t$  is most likely not the same for quarks and for gluons. Taking  $\xi = 0$  and  $t = 0$  one is then led to compare the combinations of parton densities shown in Fig. 2, keeping in mind that the dominant values of  $x$  in the convolutions (2) are generically of order  $\xi$ . We show the CTEQ6L distributions [14] at a scale of  $\mu = 2$  GeV and remark that the situation is qualitatively similar for the starting scale  $\mu = 1.3$  GeV of the CTEQ6L parameterization.

The dominance of gluons over sea quarks in the case of  $\phi$  production is so pronounced that, despite the simplicity of our estimate, we will in the following neglect  $\mathcal{F}^s$ . For the  $\rho$  channel the estimate suggests that gluons become important already at moderately small  $x_B$ . This expectation will now be confronted with data.

5. With the approximations discussed so far, the ratio of  $\phi$  and  $\rho$  production cross sections is

$$\frac{\sigma_\phi}{\sigma_\rho} \approx \frac{2}{9} \frac{|g_\rho|^2}{|g_\rho|^2 + 2|g_\rho||q_\rho| \cos \alpha + |q_\rho|^2} \quad (7)$$

with

$$\begin{aligned} |g_\rho|^2 &= \int dt \left| \frac{3}{4} \mathcal{H}^g \right|^2, & |q_\rho|^2 &= \int dt \left| \frac{2}{3} \mathcal{H}^u + \frac{1}{3} \mathcal{H}^d \right|^2, \\ |g_\rho||q_\rho| \cos \alpha &= \text{Re} \int dt \frac{3}{4} \mathcal{H}^g \left( \frac{2}{3} \mathcal{H}^u + \frac{1}{3} \mathcal{H}^d \right)^*. \end{aligned} \quad (8)$$

$\alpha$  may be regarded as the “average phase” between gluon and quark amplitudes, where the “average” is over  $t$ .

Preliminary data from HERMES [15] on  $\sigma_\phi/\sigma_\rho$  are shown in Fig. 3, together with results at very small  $x_B$  from ZEUS and H1 [16, 17, 18]. The two HERMES points with  $Q^2 = 2.46$  GeV $^2$  and 3.5 GeV $^2$  respectively correspond to  $x_B = 0.09$  and  $x_B = 0.13$ , with cross

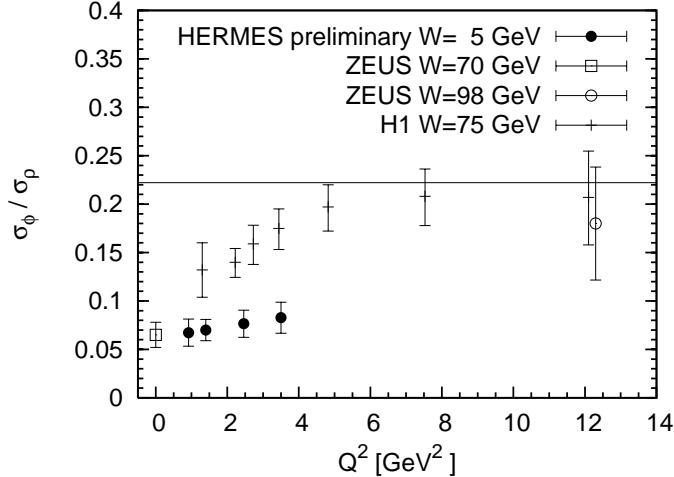


Figure 3: Data for the cross section ratio  $\sigma_\phi/\sigma_\rho$ . Statistical and systematic errors given in [17, 18] are added in quadrature. The horizontal line indicates the value  $2/9$  corresponding to a negligible quark exchange contribution in both  $\phi$  and  $\rho$  production.

section ratios of  $0.0765 \pm 0.014$  and  $0.0827 \pm 0.016$ . In the following we will take  $\sigma_\phi/\sigma_\rho = 0.08$  for simplicity. Inverting (7) one obtains

$$|q_\rho/g_\rho| = -\cos\alpha + \sqrt{(2\sigma_\rho)/(9\sigma_\phi) - \sin^2\alpha}, \quad (9)$$

where we used that the solution with a minus instead of a plus in front of the square root is not admissible for  $\sigma_\phi/\sigma_\rho < 2/9$ . This gives

$$0.38 \leq |g_\rho/q_\rho| \leq 1.5, \quad (10)$$

with the values 0.38 and 1.5 respectively corresponding to  $\cos\alpha = -1$  and  $\cos\alpha = 1$ . We remark that with current models of generalized parton distributions one typically finds that the convolution integrals (2) tend to be dominated by their imaginary parts and that these are positive, so that one may regard values of  $\cos\alpha$  near 1 as more likely. Our result (10) thus agrees rather well with the simple estimate one can obtain from Fig. 2.

6. It is well known that for vector meson production at  $Q^2$  of a few  $\text{GeV}^2$  there are sizeable power corrections (see [5, 7] and references therein). In particular, the leading approximation in  $1/Q$  cannot account for the normalization of the meson production cross section. Some but not all of the power corrections will partially cancel in the cross section ratio for  $\phi$  and  $\rho$  production.

The H1 and ZEUS electroproduction data in Fig. 3 show a clear increase of  $\sigma_\phi/\sigma_\rho$  with  $Q^2$  in a region of  $x_B$  where one would expect both  $\phi$  and  $\rho$  production to be clearly dominated by gluon exchange. It is tempting to ascribe this to an additional suppression of  $\sigma_\phi$  compared to  $\sigma_\rho$  at moderate  $Q^2$  due to effects of the strange quark mass in the quark loop of the hard scattering graphs (see Fig. 1). There is another effect contributing to this trend which already appears at leading order in  $1/Q$  and  $\alpha_S$ . In this approximation the cross section is proportional

to the square of the integral  $\int_0^1 dz z^{-1}(1-z)^{-1}\phi(z)$  over the meson distribution amplitude, where  $z$  is the momentum fraction of the quark. The above trend will thus be enhanced if at lower factorization scales  $Q^2$  the distribution amplitude is narrower for the  $\phi$  than for the  $\rho$  and only evolves to a similar shape for the two mesons with increasing  $Q^2$ .

If a similar suppression of  $\sigma_\phi$  compared with  $\sigma_\rho$  also takes place in the kinematics of the HERMES measurement, then the right-hand-side of (7) must be multiplied with a correction factor below 1, leading to an even larger estimate for  $|g_\rho/q_\rho|$  than in (10).

7. The leading approximation in  $1/Q$  predicts longitudinal polarization of both the virtual photon and the produced vector meson [9]. Experimentally one finds that both in  $\rho$  and in  $\phi$  production the ratio  $R = \sigma_L/\sigma_T$  of cross sections for longitudinal and transverse photon polarization is not very large for  $Q^2$  of a few  $\text{GeV}^2$ . This is another example that  $1/Q$  suppressed effects are not entirely negligible in this kinematic region. In the relation (7) we should more precisely have the ratio  $\sigma_{L\phi}/\sigma_{L\rho}$  of longitudinal cross sections on the left-hand-side. Preliminary HERMES data [19, 20, 21] suggest that  $R_\phi$  may be somewhat smaller than  $R_\rho$  in the kinematics of the HERMES points in Fig. 3. This would correspond to  $\sigma_{L\phi}/\sigma_{L\rho} < \sigma_\phi/\sigma_\rho$  and again lead to a larger estimate for  $|g_\rho/q_\rho|$  than in (10).

8. In summary, we find that under rather weak assumptions the HERMES data on the ratio of  $\phi$  and  $\rho$  electroproduction cross sections indicates a substantial contribution from gluon exchange in the  $\rho$  channel for  $x_B$  around 0.1 and  $Q^2$  of a few  $\text{GeV}^2$ .

This conclusion is in contrast with the results of [22], shown for the kinematics of the HERMES measurement in [23], where the gluon contribution to the cross section was estimated to be quite small. We cannot fully resolve this discrepancy here, but remark that for the gluon exchange contribution to  $\rho$  production Ref. [22] used the calculation of [24], which was performed for the limit of very small  $x_B$ . It is difficult to assess the reliability of extrapolating the results of [24] to  $x_B \sim 0.1$ . A model calculation of both quark and gluon contributions in  $\rho$  production based on the convolutions in (2) is under way [25].

**Acknowledgments.** We thank E.-C. Aschenauer, A. Borissov and W.-D. Nowak for valuable discussions or correspondence. A.V. is supported by RFBR grants 04-02-16445 and 03-02-17291 and by the Heisenberg-Landau program.

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